

Charged Lifshitz black hole and probed Lorentz-violation fermions from holography

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Abstract

We analytically obtain a new charged Lifshitz solution by adding a non-relativistic Maxwell field in Hořava-Lifshitz gravity. The black hole exhibits an anisotropic scaling between space and time (Lifshitz scaling) in the UV limit while in the IR limit, the Lorentz invariance is recovered. We introduce the probed Lorentz-violation fermions into the background and holographically investigate the spectral properties of the dual Fermionic operator. The Lorentz-violation of fermions will enhance the peak and corresponds larger fermi momentum, which compensates the non-relativistic bulk effect of the dynamical exponent. For a fixed z , when the Lorentz-violation of fermions increases to a critical value, the behavior of the low energy excitation goes from a non-Fermi liquid type to a Fermi liquid type, which implies a kind of phase transition.

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I. INTRODUCTION

The Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence[1–4] is opening a new avenue to disclose many different strongly interacting systems. The correspondence makes a connection between gravitational theories on AdS spacetime and a quantum field theory(QFT) that lives on the conformal boundary of the AdS spacetime. Due to its remarkable feature that it connects a strong coupling QFT to a weak coupling gravitational theory, this useful tool has been attracting considerable interest in studying strongly coupled physics, especially, the possible applications to condensed matter physics(see [5–7] for reviews).

The AdS/CFT duality is firstly generated in [8] to study a boundary theories with dynamical critical exponents which present the dynamical scaling

$$t \rightarrow \lambda^z t, \ x \rightarrow \lambda x \quad \text{with} \quad z \neq 1 \quad (1)$$

instead of the scale invariance with $z = 1$. It is pointed out that the background geometry which is holographically applied to study the nonrelativistic QFT with dynamical (Lifshitz) scaling (1) is given by the Lifshitz spacetime

$$ds^2 = -r^{2z} dt^2 + \frac{dr^2}{r^2} + r^2 dx_i^2, \quad (2)$$

where the spacial index i runs from 1 to $D - 2$ and we have set the radius of curvature to be unit.

In [8], though the dual QFT is nonrelativistic with Lifshitz scaling, the bulk gravity is relativistic which are invariant under the full spacetime diffeomorphism and it has to couple with matter to generalize the Lifshitz geometry. This situation is improved by the Hořava-Lifshitz(HL) gravity [9, 10] which is based on the perspective that Lorentz symmetry should appear as an emergent symmetry at long distances, but can be fundamentally absent at short ones [11, 12], and it is a non-relativistic power-counting renormalizable theory of gravitation. Since the HL gravity is itself anisotropic between space and time, it is natural to expect that the HL gravity provides a minimal holographic dual for nonrelativistic Lifshitz-type QFT. This proposal has been realized and carefully studied in [13–15]. Especially, it was addressed in [13] that the Lifshitz spacetime is a vacuum solution to the HL gravity. Analytical vacuum solution with asymptotically Lifshitz geometry to the gravity was also found in [16, 17]. The

holographic applications in the framework of the HL gravity were constructed (for instance, holographic superconductor models in the HL gravity were studied in [18, 19]).

In this paper, as a first step, we couple the gravitational action of HL gravity with a $U(1)$ gauge field whose Lagrangian is consistent with the symmetry of HL gravity. We analytically solve the equations of motion for the gravity and the gauge field, and obtain a new charged Lifshitz black hole solution to the coupled theory. As we will show later, in the UV limit, our charged solution behaves as the geometry (2) while at the IR limit, the Lorentz invariance (with $z = 1$) is restored and the geometry is $AdS_2 \times \mathbb{R}^{D-2}$.

The asymptotical characters of our solution motivate us to introduce the Lorentz-violation fermions in the charged HL background and holographically study the properties of fermionic spectral. On the other hand, holography has been widely applied to explore the strongly correlated fermionic system. In the pioneer works [20–24] on this topic, the authors proposed that by studying the bulk Dirac equation in the RN-AdS black hole with the ingoing behavior at the horizon, the fermionic correlator can be extracted at the AdS boundary. The proposal helps us understanding better the mysterious behaviors of the existing (non-)Fermi liquid. It is also inspiring more and more efforts to be involved in the related topics¹. However, the previous works usually started by adding relativistic fermions into the relativistic bulk theory. Since the anisotropic scaling (1) is built in by construction in the HL gravity, it is natural to expect the coupling matter exhibits the same scaling in the same footing. Then we should ask what are the results of introducing Lorentz-violation fermions as a probe. Aiming to answer this question is our another motivation to introduce the probed non-relativistic fermions into the non-relativistic HL gravity and study the effect of the Lorentz-violation on the Fermionic spectral function.

Our results show that, different from effect of dynamical exponent which suppresses the peak of the fermionic spectral function[29, 36], the Lorentz-violation of fermions will enhance the peak and make the fermi momentum larger to compensate the non-relativistic bulk effect. Also, for fixed z , small Lorentz-violation of fermions corresponds the low energy

¹ The generalization of holographic fermions obtain many remarkable progress, such as the effects of different bulk theory on the fermionic correlation[25–33], holographic non-relativistic fermionic fixed points by imposing the Lorentz violating boundary condition[34–36], phase transition from the Fermi liquid to non-Fermi liquid and also to the Mott insulating phase due to the interaction between Dirac field and gauge field[37–42] and the fermionic spectral function effected by lattice effects[43–45], etc..

excitation as non-Fermi liquid with nonlinear dispersion relation, while once the Lorentz-violation becomes larger than a critical value, the low energy excitation always behaves as Fermi liquid with linear dispersion relation. This may imply that in the non-relativistic HL gravity, it is more natural to introduce Lorentz-violation of fermions to dually describe a fermionic system whose low energy excitation is Fermi liquid.

The remaining of this paper is organized as follow. In section II, we get a charged black hole solution by solving the coupled system of non-relativistic gauge field and HL gravitational theory. We show the holographic setup by introducing the Lorentz-violation fermions into the coupled background sector in section III. Then in section IV, we exhibit the numerical result of the Fermi momentum and dispersion relation by analyzing the spectral function. Section V contributes our conclusion and discussion.

II. ANALYTICAL CHARGED LIFSHITZ SOLUTION FROM HORAVA-LIFSHITZ GRAVITY

It was studied in Ref. [13] that the HL gravity can provide solution dual to Lifshitz-type field theories, where the following gravitational action is proposed

$$S_g = \frac{1}{2\kappa^2} \int dt dr d^d x \sqrt{-G} (K_{ab} K^{ab} - \lambda K^2 + \beta(R - 2\Lambda) + \frac{\alpha^2}{2} \frac{\nabla_a N \nabla^a N}{N^2}). \quad (3)$$

It will return to Einstein gravity, when $\lambda = \beta = 1$ and $\alpha = 0$. In Ref. [13], the authors found that: For the requirement of stability and perturbative unitarity around flat spacetime, constraints should be imposed on the couplings, i.e., $\beta > 0$, $\alpha \leq \frac{2\beta d}{d-1}$ and $\lambda \geq 1$ or $\lambda^2 \leq \frac{1}{d-1}$ for $\Lambda = 0$. When $\Lambda < 0$, if we take

$$\Lambda = -\frac{(d+z-1)(d+z)}{2}, \text{ and } \alpha^2 = \frac{2\beta(z-1)}{z}, \quad (4)$$

the Lifshitz metric (2) is also a vacuum solution of the equation (3). In our paper, we will use the condition of $\Lambda < 0$. Besides, the authors worked with the Arnowitt-Deser-Misner metric ansatz,

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt). \quad (5)$$

Here, N , N^i and g_{ij} are the lapse function, the shift vector and the metric of the spacelike hypersurface, respectively. Thus, in the action, one has the formulae $\sqrt{-G} = \sqrt{g}N$, $K_{ab} = \frac{1}{2N}(\partial_t g_{ab} - \nabla_a N_b - \nabla_b N_a)$ and $K = g^{ab} K_{ab}$, where R is the scalar curvature of the metric

g_{ab} . Later, the authors of [16] analytically studied the neutral Lifshitz black hole solutions to the equations of motion deduced from the above action.

We intend to explore the possible charged black hole solutions in HL gravity. We consider Lorentz-violated electromagnetism field coupling to the HL gravity with the action [46]

$$S_m = -\frac{1}{2\kappa^2} \int dt dr d^d x \sqrt{g} N \left(\frac{2}{N^2} g^{ij} (F_{0i} - F_{ki} N^k) (F_{0j} - F_{\ell j} N^\ell) - F_{ij} F^{ij} - \beta_0 - \beta_1 a_i B^i - \beta_2 B_i B^i \right), \quad (6)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $B^i = \frac{1}{2} \frac{\epsilon^{ijk}}{\sqrt{g}} F_{jk}$ with ϵ^{ijk} the Levi-Civita symbol. Then the total action of the background we will take account into is

$$S_t = S_g + S_m. \quad (7)$$

We consider the electromagnetic field with the only non-vanishing component $A_t(r)$ and also $\beta_\mu = 0 (\mu = 0, 1, 2)$. Subsequently, the Maxwell equation reads as

$$\partial_r (\sqrt{g} N F^{rt}) = 0, \quad (8)$$

which gives us the solution

$$F^{rt} = \frac{Q_e}{\sqrt{g} N} \quad (9)$$

with the integral constant Q_e , which can be interpreted as the charge of the Lifshitz black hole as we can see latter.

In order to obtain the complete solutions to the equations of motion given by the action (7), we will borrow the analytical process present in Ref. [16]. To proceed, we set the metric components in (5) as

$$N = e^{2f(r)}, \quad g_{rr} = \frac{1}{e^{2h(r)}}, \quad g_{ii} = e^{2l(r)}, \quad N_a = 0. \quad (10)$$

By substituting the formulas in (10) into the action (7), we obtain

$$S_t = \frac{d\beta v}{2\kappa^2} \int dr e^{dl+h+f} \left((d-1)l'^2 + 2l'f' + \delta f'^2 - \frac{2\Lambda}{d} e^{-2h} - \frac{2Q_e^2}{d\beta} e^{-2h-2dl} \right), \quad (11)$$

where $v = \int dt d^d x$ and

$$\delta = \frac{\alpha^2}{2d\beta}. \quad (12)$$

Considering the above S_t as a one dimensional action of the functions $f(r)$, $l(r)$ and $h(r)$ and varying the action, we get two independent equations

$$(e^{dl+h+f}(l' + \delta f'))' = -\frac{2Q_e^2}{d\beta}e^{f-h-dl} - \frac{2\Lambda}{d}e^{f-h+dl}, \quad (13)$$

$$(e^{dl+h+f}((d-1)l' + f'))' = -2\Lambda e^{f-h+dl}, \quad (14)$$

which can be further simplified as

$$(e^{dl+h+f}((d\delta - 1)f' + l'))' = -\frac{2Q_e^2}{\beta}e^{f-h-dl}. \quad (15)$$

It seems not so direct to find the solutions of the three functions $f(r)$, $l(r)$ and $h(r)$, but we can assume the following forms

$$f = z \ln r + \frac{1}{2} \ln \xi(r), \quad h = \ln r + \frac{1}{2} \ln \xi(r), \quad l = \ln r, \quad (16)$$

so that the metric becomes

$$ds^2 = -r^{2z}\xi(r)dt^2 + \frac{dr^2}{r^2\xi(r)} + r^2dx_i^2. \quad (17)$$

The question now reduces to find the solution of $\xi(r)$. Inserting the formulas (16) into equation (15), gives us the equation of motion for $\xi(r)$

$$\left(\frac{1}{2z}r^{d+z+1}\xi'(r)\right)' = -\frac{2Q_e^2}{\beta}r^{z-1-d}. \quad (18)$$

It is obvious that if we take the constraints (4), then we can get the solution of $\xi(r)$

$$\xi(r) = 1 - \frac{M}{r^{d+z}} + \frac{2zQ_e^2}{d\beta(d-z)}\frac{1}{r^{2d}}, \quad (19)$$

where M and Q_e are integral constants which can be understood as the mass and charge of the Lifshitz black hole. At the horizon r_0 , $\xi(r_0)$ should be zero, which implies that $M = r_0^{d+z} + \frac{2zQ_e^2}{d\beta(d-z)}\frac{1}{r_0^{d-z}}$. Substituting the solved metric into (8), we can fix the Maxwell field

$$A_t = \mu \left(1 - \left(\frac{r_0}{r}\right)^{d+z-2}\right) \quad \text{with} \quad \mu = \frac{Q_e}{d+z-2}. \quad (20)$$

The Hawking temperature of the charged black brane is

$$T = \frac{(d+z)r_0^z - \frac{2z}{d\beta}Q_e^2r_0^{z-2d}}{4\pi}. \quad (21)$$

To proceed, we will give some comments on our black hole solution.

Firstly, the solutions (17), (19) and (20) are a new charged Lifshitz solution which is different from the solution proposed in [47–52]. However, it also recovers AdS RN black hole solution when $z = 1$. It is worthy to point out that from the expression of δ in equation (12), $\alpha = 0$ will force us to keep $z = 1$ which is the AdS RN solution. Thus we can only expect the above Lifshitz solution when $\alpha \neq 0$, which also happens in neutral case studied in [16].

Secondly, from the expression of temperature (21), it is straightforward that the extremal case is fulfilled when

$$Q_e = \sqrt{\frac{d\beta(d+z)}{2z}} r_0^d, \quad \text{or} \quad \mu = \frac{1}{(d+z-2)} \sqrt{\frac{d\beta(d+z)}{2z}} r_0^d. \quad (22)$$

Then it is explicit that in order to have real chemical potential, the condition $\beta > 0$ should be satisfied. Furthermore, recalling the expression of δ in equation (12), we should have the range of the Lifshitz exponent as $z > 1$ with $\alpha \neq 0$.

Thirdly, we will check the asymptotical behavior of our solution. In asymptotical boundary $r \rightarrow \infty$, since $\xi(r \rightarrow \infty) \rightarrow 1$, so that the metric (17) reduces to the asymptotical Lifshitz metric (2). While, near the horizon, the geometry is a bit subtle. For simplicity, we will set the horizon $r_0 = 1$ in the following discussion, then the redshift factor at zero temperature can be rewritten as

$$\xi(r) = 1 - \frac{2d}{(d-z)r^{d+z}} + \frac{d+z}{(d-z)r^{2d}}. \quad (23)$$

whose behavior near the horizon is $\xi(r) \simeq 2d(d+z)(r-1)^2 = \frac{1}{L_2^2}(r-1)^2$ with $L_2 = 1/\sqrt{2d(d+z)}$. Therefore, at the zero temperature, the near horizon geometry is $AdS_2 \times \mathbb{R}^d$ with the AdS radius L_2 . This is more explicit by considering the transformation

$$r-1 = \lambda \frac{L_2^2}{\varsigma}, \quad t = \lambda^{-1} \tau, \quad \lambda \rightarrow 0 \text{ with } \varsigma, \tau \text{ finite}. \quad (24)$$

Consequently, the metric (17) can be deduced into the $AdS_2 \times \mathbb{R}^d$ form

$$ds^2 = \frac{L_2^2}{\varsigma^2} (-d\tau^2 + d\varsigma^2) + dx_i^2 \quad (25)$$

and the gauge field is

$$A_\tau = \frac{(d+z-2)\mu L_2^2}{\varsigma}. \quad (26)$$

Since the near horizon geometry is $AdS_2 \times \mathbb{R}^d$ and the solution is Lifshitz in UV limit, we can use the Lifshitz holography [8] to study its application in strongly coupled physics.

In the next section, as an attempt, we will study the fermionic spectral of the operator dual to the probed fermions in this background .

III. HOLOGRAPHIC SETUP OF LORENTZ VIOLATION FERMIONS

We will introduce the fermions into the above theory as a probe and apply the holography to study the fermionic correlation. Because the HL gravity is a non-relativistic bulk theory, it is physically more nature to consider the the Lorentz-violation action for the probed fermionic sector [53]

$$S = \int d^{d+2}x \sqrt{g} \bar{\psi} (\Gamma^a e_a^\mu (\nabla_\mu - iqA_\mu) + M) \psi, \quad (27)$$

with

$$\begin{aligned} \Gamma^a &= \gamma^a - c_{\mu\nu} e^{\mu\nu} e_b^\mu \gamma^b - d_{\mu\nu} e^{\mu\nu} e_b^\mu \gamma^5 \gamma^b - e_\mu e^{a\mu} - if_\mu e^{a\mu} - \frac{1}{2} g_{\lambda\mu\nu} e^{\mu\nu} e_b^\lambda e_c^\mu \gamma^{bc}, \\ M &= m_0 + im_5 \gamma^5 + m_\mu e_a^\mu \gamma^a + b_\mu e_a^\mu \gamma^5 \gamma^a + \frac{1}{2} H_{\mu\nu} e_a^\mu e_b^\nu \gamma^{ab}, \\ \nabla_\mu &= \partial_\mu - \frac{1}{4} \omega_{\mu ab} \gamma^{ab}, \quad \gamma^{ab} = \frac{1}{2} [\gamma^a, \gamma^b], \end{aligned} \quad (28)$$

The first term in Γ^a and the first two terms in M respectively lead to the usual Lorentz-invariant kinetic term and mass term for the Dirac field. In order to take Lorentz-violation Γ^a and M , we will choose the setting of parameters as $c_{\mu\nu} = \rho h_{\mu\nu}$ where $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$ and $n_\mu = (-N, 0, 0, 0)$, $H_{tr} = H_{rt} = i\eta$, $m_0 = -m = const.$, and the other coefficients vanishes.

From the action (27), the equation of motion for Dirac field is

$$(\Gamma^a e_a^\mu (\nabla_\mu - iqA_\mu) + M) \psi = 0. \quad (29)$$

Redefining the field as

$$\psi = (-gg^{rr})^{\frac{1}{4}} \int d\omega dk e^{-i\omega t + ik_i x^i} \phi, \quad (30)$$

and setting $\phi = (\phi_1, \phi_2)^T$ with ϕ_I the two-component spinors, then choosing $k_i = k\delta_x^i$ because of the rotation symmetry in the spatial directions, we reduce the Dirac equation (29) into the form

$$\left(\frac{\sqrt{g_{xx}}}{\sqrt{g_{rr}}} \partial_r + m\sigma^3 \sqrt{g_{xx}} \right) \phi_I = \left(\frac{\sqrt{g_{xx}}}{\sqrt{g_{tt}}} (\omega + qA_t) i\sigma^2 \mp k\sigma^1 - \frac{\eta}{2a} \sigma^1 \sqrt{\frac{g_{xx}}{g_{tt}g_{rr}}} \right) \phi_I, \quad (31)$$

with $a = (1 - (d + 1)\rho)$ after we choose the gamma matrices as [54]

$$\Gamma^r = \begin{pmatrix} -\sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}, \quad \Gamma^t = \begin{pmatrix} i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix}, \quad \Gamma^x = \begin{pmatrix} -\sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}. \quad (32)$$

Further setting $\phi_I = (X_I, Y_I)^T$, we can divide the above equation (31) into two equations

$$\frac{\sqrt{g_{xx}}}{\sqrt{g_{rr}}} \partial_r X_I + m \sqrt{g_{xx}} X_I = \frac{\sqrt{g_{xx}}}{\sqrt{g_{tt}}} (\omega + q A_t) Y_I \mp k Y_I - \eta \sqrt{\frac{g_{xx}}{g_{tt} g_{rr}}} Y_I, \quad (33)$$

$$\frac{\sqrt{g_{xx}}}{\sqrt{g_{rr}}} \partial_r Y_I - m \sqrt{g_{xx}} Y_I = -\frac{\sqrt{g_{xx}}}{\sqrt{g_{tt}}} (\omega + q A_t) X_I \mp k X_I - \eta \sqrt{\frac{g_{xx}}{g_{tt} g_{rr}}} X_I. \quad (34)$$

Not that we have shifted $\frac{\eta}{2a} \rightarrow \eta$ in the equations. Later, we will use η to denote the strength of the Lorentz-violation of the fermions, and $\eta = 0$ means that the fermions are Lorentz-invariance. With the above equations in hands, we further introduce the new ratios $\chi_I = \frac{X_I}{Y_I}$. Hereafter, the Dirac equation can be reduced into

$$\begin{aligned} (r^2 \sqrt{\xi(r)} \partial_r + 2mr) \chi_I &= \frac{1}{r^{z-1} \sqrt{\xi(r)}} (\omega + q A_t) + (-1)^I k - \frac{\eta}{r^{z-2}} \\ &+ \left(\frac{1}{r^{z-1} \sqrt{\xi(r)}} (\omega + q A_t) - (-1)^I k + \frac{\eta}{r^{z-2}} \right) \chi_I^2. \end{aligned} \quad (35)$$

The boundary condition of χ_I near the horizon $r \rightarrow 1$ is

$$\chi = i. \quad (36)$$

UV behavior

Now we move on to study the behavior of fermion field near boundary $r \rightarrow \infty$. In this limit, the metric (17) becomes

$$g_{tt} \rightarrow -r^{2z}, \quad g_{rr} \rightarrow \frac{1}{r^2}, \quad g_{xx} = r^2, \quad (37)$$

with $z > 1$ and the equation (31) becomes

$$(r^2 \partial_r + mr \sigma^3 - r^{1-z} (\omega + q A_t) i \sigma^2 - (-1)^I k \sigma^1 - \eta r^{2-z} \sigma^1) \phi_I = 0. \quad (38)$$

of which the leading order² is

$$(\partial_r + \frac{m \sigma^3}{r}) \phi_I = 0, \quad (39)$$

² When $z \rightarrow 1$, the leading order is too stiff to say (39) is the leading order of equation (38). In fact, however, in our numerical calculation, we always choose $z = 1 + \epsilon$ where ϵ is not a small number, it cannot be ignored. So when $r \rightarrow \infty$, it will be general to choose the leading order like equation (39).

and its solution near the boundary is

$$\phi_I = a_I r^{-m} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b_I r^m \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{with } r \rightarrow \infty. \quad (40)$$

As it is addressed in [24], choosing the normal boundary term

$$S_{bdy} = \frac{i}{2} \int_{\partial \mathcal{M}} dt d^d x \sqrt{-g g^{rr}} \bar{\phi} \phi, \quad (41)$$

we can suppose that the source and the response in (40) are related by

$$a_I \begin{pmatrix} 1 \\ 0 \end{pmatrix} = S b_I \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (42)$$

and the boundary Green's function $G(\omega, k)$ is

$$G = -i S \gamma^0. \quad (43)$$

Recalling the definition of $\chi_I = \frac{X_I}{Y_I}$, we can rewrite the Green function as

$$G(\omega, k) = \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} = \lim_{r \rightarrow \infty} r^{2m} \begin{pmatrix} \chi_1 & 0 \\ 0 & \chi_2 \end{pmatrix}. \quad (44)$$

IR behavior

In near horizon geometry $AdS_2 \times \mathbb{R}^d$, the Dirac equation (31) in the zero frequency limit, or low energy limit, can be rewritten as

$$\varsigma \partial_\varsigma \phi_I - (m L_2 \sigma^3 - i(d+z-2)q\mu L_2^2 \sigma^2 - (-1)^I L_2 k \sigma^1 - \frac{\eta \varsigma^2 \sigma^1}{L_2}) \phi_I = 0, \quad (45)$$

where $\Gamma^\varsigma = -\Gamma^r$ and others are the same as (32) when we reflect the change between the radial coordinate r to the coordinate ς . Moreover, near the AdS_2 boundary with $\varsigma \rightarrow 0$, the leading term of the above equation is

$$\varsigma \partial_\varsigma \phi_I = \begin{pmatrix} m L_2 & -(d+z-2)q\mu L_2^2 - (-1)^I L_2 k \\ (d+z-2)q\mu L_2^2 - (-1)^I L_2 k & -m L_2 \end{pmatrix} \phi_I, \quad (46)$$

which gives us the leading behaviour of ϕ_I as

$$\phi_I = b_I^{(0)} v_- \varsigma^{-\nu(k)} + a_I^{(0)} v_+ \varsigma^{\nu(k)}, \quad (47)$$

where v_{\pm} and $\pm\nu(k)$ are real eigenvectors and eigenvalues of the matrix in the right side of equation (46) with the form

$$\nu(k) = \sqrt{(m^2 + k^2)L_2^2 - (d + z - 2)^2 q^2 \mu^2 L_2^4}. \quad (48)$$

Subsequently, according to the discussion in [22], in the IR CFT the conformal dimension of the operator \mathbf{O}_k will be $\delta_k = 1/2 + \nu(k)$, which is dependent on the boundary dimension and Lifshitz dynamical critical exponent because they have information in the IR geometry.

IV. RESULTS OF FERMI SURFACE AND DISPERSION RELATION

In this section, we will numerically solve the Dirac equation (35) to investigate the momentum of Fermi surface and the dispersion relation of the excitation near the Fermi surface. In the previous works[29, 36], the authors claimed that as the Lifshitz exponent z increases, the peak of the Fermi-like peak suppresses and for large enough z , the peak is too small to be observed. In our model, the same phenomena is observed. Thus, in our study, we will fix a small $z = 1.02$ and mainly focus on the effects of the Lorentz violation parameter η on the holographic fermionic systems. Meanwhile, we will fix the mass of fermions $m = 0$ and charge $q = 1$ in the $3 + 1(d = 2)$ dimensional bulk geometry³.

Then from equation (35), the Green function satisfies the following symmetry

$$G_1(\omega, k) = G_2(\omega, -k), \quad G_I(\omega, k) = -1/G_I(\omega, -k), \quad (49)$$

so that without loss of generality, we can only study the component G_2 with positive k for the Fermi surface and dispersion relation. Note that we will study in zero temperature with μ having the form in equation (22).

³ Note that effect of dimension of theory and fermionic mass as well as charge on holographic fermions has been carefully investigated in some gravitational theories[26, 36].

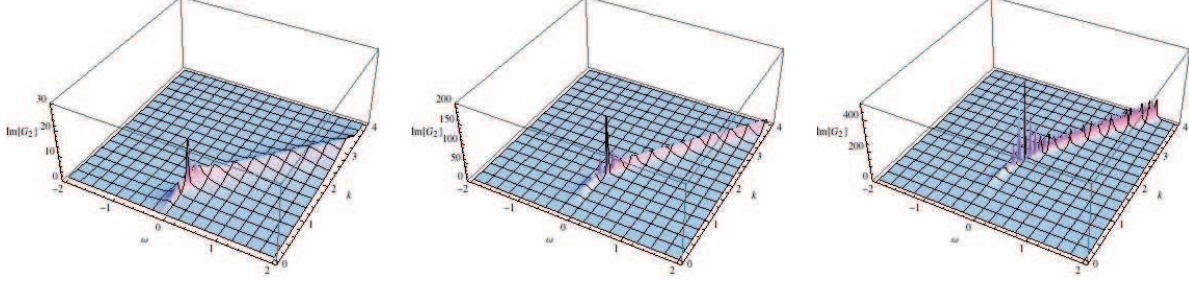


FIG. 1: The Green function with $\eta = 0$ (left), $\eta = 0.03$ (middle), $\eta = 0.05$ (right).

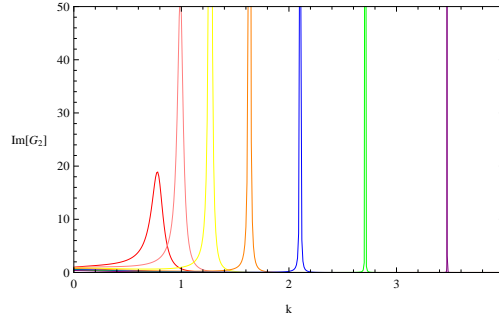


FIG. 2: Green function changing as the momentum with tiny $\omega = 10^{-5}$ for different η . From left to right, we increase η from 0 to 0.06 with the step 0.01.

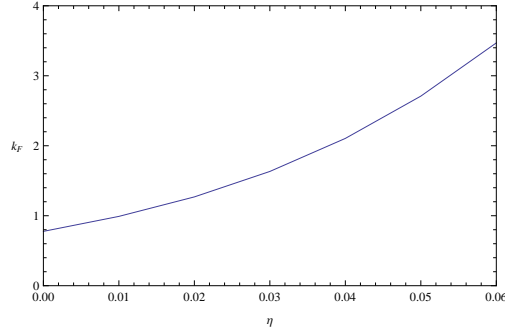


FIG. 3: The change of Fermi momentum k_F as η .

In figure 1, we show how the Green function G_2 behaves as the momentum and frequency for different η . The left plot corresponds to $z = 1.02$ and $\eta = 0$, which shows a sharp Fermi-like peak at zero frequency implying a Fermi surface. And, this peak becomes sharper and its location of momentum is larger when we increase the Lorentz violation parameter η , as shown in the middle and right plots. This phenomena is more obvious in figure 2 where we fix a tiny frequency $\omega = 10^{-5}$ and plot the behavior of G_2 by changing the momentum. It

means that η compensates the effect of the Lifshitz exponent which suppresses the height of peak. From this perspective, this observe may explain why it is more reasonable to introduce Lorentz violation fermions to our black hole solution to holographically study the spectral in the boundary theory since we always expect sharp peak at $\omega = 0$ to define the Fermi-like surface. From figure 2, we can roughly read off the location of the peak, which denotes the

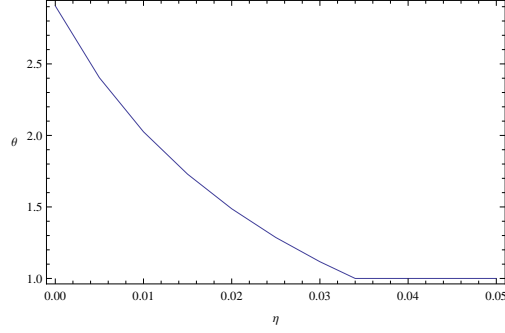


FIG. 4: The relation between θ and η .

Fermi momentum k_F of the Fermi surface. By more careful calculation, we plot the related values of fermi momentum k_F for different η in figure 3, which shows us that k_F increases as the Lorentz violation of Fermions is enhanced.

After we determine the fermi momentum, we can directly calculate the dispersion relation which is defined as [22]

$$\tilde{\omega}(\tilde{k}) \propto \tilde{k}^\theta, \quad \text{with} \quad \theta = \begin{cases} \frac{1}{2\nu(k_F)} & \text{for } \nu(k_F) < \frac{1}{2} \\ 1 & \text{for } \nu(k_F) > \frac{1}{2} \end{cases}. \quad (50)$$

where $\tilde{k} = k - k_F$ and $\tilde{\omega}$ denote the shift away from the fermi surface with $(k = k_F, \omega \rightarrow 0)$, and $\nu(k)$ is defined in equation (48). Substituting the determined Fermi momentum into (50), we can compute the exponent θ of the dispersion relation, which is shown in figure 4. When η is smaller than 0.034, the exponent θ decreases with the increasing of η and it is always higher than unit, which implies that the low energy excitation is non-Fermi Liquid. On the other side, when η is larger than 0.034, the low energy excitation may behave as Fermi Liquid because θ keeps at unit. Our result implies that when the parameter of Lorentz violation of fermions goes strong enough, there may be a phase transition from non-Fermi liquid to Fermi liquid in boundary fermionic system. Thus, in order to holographically describe the Fermi liquid in non-relativistic HL gravity, it may be more reasonable to introduce Lorentz-violation fermions.

V. CONCLUSION AND DISCUSSION

In this paper, we analytically solved the equations of motion for the coupling system between non-relativistic gauge field and HL gravitational action. We obtained a new charged Lifshitz black hole solution, which in the UV limit behaves the Lifshitz geometry (2) with a dynamical scaling between space and time, while at zero temperature, the near horizon geometry is $AdS_2 \times \mathbb{R}^{D-2}$. Here we did not explore more detailed features of our new black hole, such as thermodynamics and various stabilities, etc., but we will present this study somewhere else in the near future.

Then, we holographically studied the properties of the fermionic sector dual to the charged HL gravity theory, by introducing the probed Lorentz-violation Fermions into the background. We fixed $z = 1.02$ and mainly focused on the effect of the Lorentz-violation feature on the Fermi momentum and the dispersion relation. We found that for stronger Lorentz-violation of the fermions, the peak in the spectral function, which is suppressed by the dynamical exponent, is enhanced and the Fermi momentum is larger. Meanwhile, the stronger violation makes the exponent of dispersion relation lower until it reaches a critical value $\eta \simeq 0.034$, the dispersion relation is kept linear even we further enhance the violation, which means that at the critical value $\eta \simeq 0.034$, the type of low energy excitation will translated from non-Fermi liquid to Fermi Liquid. We also argued that in order to describe a Fermi type Liquid of the low energy excitation, it is more reasonable to add the Lorentz-violation fermions in the non-relativistic charged HL gravity.

Note that in the fermionic sector we studied, we chose the boundary condition (40) for the fermionic field. Since all fields in our system are non-relativistic, we have strong motivation to choose the Lorentz violation boundary term into bulk fermionic action

$$S_{bdy} = \frac{1}{2} \int_{\partial\mathcal{M}} d^3x \sqrt{-g} g^{rr} \bar{\phi} \Gamma^1 \Gamma^2 \phi . \quad (51)$$

With the above boundary condition, it was first addressed in [34] that the spectral function of the dual holographic fermionic system showed a flat band of gapless excitation. Then we follow the discussion of [34], in this case, the retarded Green function is a kind of recombination of Green functions of the relativistic fixed point as

$$G_R = \begin{pmatrix} \frac{2G_1 G_2}{G_1 + G_2} & \frac{G_1 - G_2}{G_1 + G_2} \\ \frac{G_1 - G_2}{G_1 + G_2} & \frac{-2}{G_1 + G_2} \end{pmatrix} \quad (52)$$

and due to the symmetry (49), the spectral function is

$$A_{NR}(\omega, k) = \text{Im Tr}[G_R] = \text{Im} \left[\frac{4G_I}{1 - G_I^2} \right] . \quad (53)$$

However, we did not find Fermi surface in this case of our system. The physics behind this phenomena calls for further research to disclose and we hope that the further study on the features of our black hole solution may give some insight.

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